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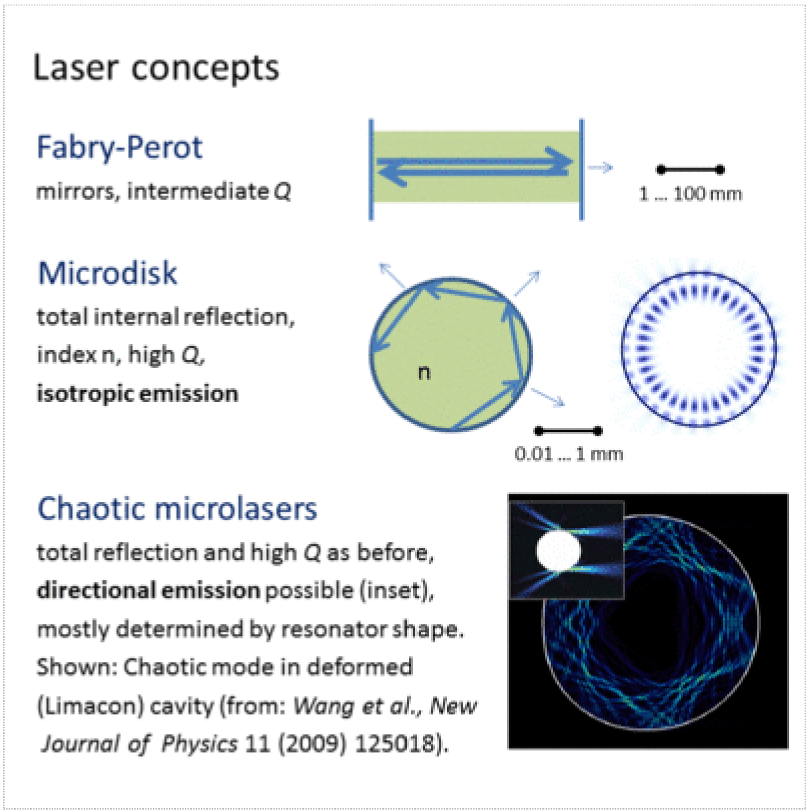

Figure 1: From Fabry-Perot to chaotic microcavity lasers.

Since the mid 1990s, **Chaotic microlasers** have been established as an alternative to the conventional and well-known Fabry-Perot lasers in the course of the ongoing miniaturization of devices. Those microdisk lasers (see for example McCall et al.(1992) or Vahala (2003)) allow one to keep high quality (or Q) factors which is important, e.g., in micro- and nanophotonic applications. Chaotic microlasers are typically realized as so-called microcavity lasers, i.e., essentially planar systems (the third dimension can be neglected) of slightly deformed disk shape, as shown in the lowest panel of Figure 1. They have to be distinguished (at least for the purpose of this article) from other microlasers such as VCSELs (vertical cavity surface emitting lasers, truly three-dimensional structures similar to a vertical Fabry-Perot set-up, with typically mid-range Q factors). Besides the quest for developing high Q microcavity lasers driven from the experimental and application-oriented point of view, microcavity lasers open actually a new venue for the established fields of chaos and quantum chaos: They represent open two-dimensional billiards systems. Therefore, the theoretical description of chaotic microlasers turns out to be very closely related to the fields of dynamical systems, chaos and quantum chaos where hard wall, closed billiards have been used as model systems for a long time. We shall see below how close the relation is - and one important difference: As light may escape by diffraction, microcavity lasers are intrinsically open systems. The hard-wall case, i.e., the closed billiards typically studied in the field of quantum chaos, is only reached in the limit of a (infinitely) large refractive index when the condition of total internal reflection is always fulfilled, and when evanescent leakage of light can be neglected.

Contents

Conventional, microdisk, and chaotic lasers

We all know the Fabry-Perot laser - light travels back and forth between two parallel mirrors and through an active medium that provides the amplification so that we can have a coherent and monochromatic light source, see top panel of Figure 1 above. How far can one down-scale this concept? It will certainly become difficult on the micrometer scale - both technically and because the much reduced optical path through the active material yields an insufficient amplification. Fortunately, the so-called microdisk lasers (McCall (1992)) provide a way out: In rotationally symmetric systems, light is confined in so-called whispering gallery modes by total internal reflection. The amplification problem is solved as expressed by the very high Q factor of these systems. The evanescent light escape is by tunneling - preserving, of course, the rotational symmetry of the system and thus leading to a uniform light output in all directions: This is not yet a useful laser because the directionality of the light emission, characteristic for all lasers, is missing.

At this point chaos comes in: In order to obtain a suitable modulation of the laser's far field, one has to break the rotational symmetry - that is, to deform the circular shape of the resonator. These systems are referred to as **chaotic microlasers** or deformed microdisk/microcavity lasers. In fact, very little deformation suffices, see the example in Figure 1, bottom right, in order to drastically change the far field. This means that the resonator shape still appears to be rather circular, and that light can still be confined in whispering-gallery-like modes such that a high Q factor is ensured. However, the rotational symmetry is sufficiently broken such that one can really obtain a rather directional far-field emission from such modes, in the example shown the emission is predominantly to the right. It corresponds to the high central peak in the far-field characteristics shown in Figure 2. The chaotic microlaser cavity shown here has the so-called Limaçon shape. It is given in polar coordinates (r, ϕ) as $r(\phi) = R_0[1 + \epsilon\cos(\phi)]$ with mean radius R_0 and deformation parameter ϵ.

From chaos to directional emission

The defining property of a laser is **coherent light emission in a certain direction**. How to fulfill this requirement with chaotic microdisk laser? And would *chaotic* not imply that the far-field characteristics is sensitive to the slightest changes in, e.g., the shape of the microcavity? In other words: How (actually: *why*) can one make a chaotic microlaser sufficiently robust to be useful for applications in future devices?

At this point it proves useful to consider our chaotic microdisk lasers as billiards for light, i.e. the light is confined just as in a usual (hard wall) billiard if the conditions for total internal reflection are fulfilled, but can in principle leave the system by refractive escape - billiards for light are open systems. Light is confined by total internal reflection as long as the angle of incidence is larger than the critical angle χ_{cr} given as $\sin \chi_{cr} = 1/n$ for a cavity of refractive index $n > 1$ in air, for smaller angles of incidence, the light is at least partially transmitted outside the microlaser according to Fresnel's laws. And whereas the angle of incidence is conserved in circular microlasers, it varies in chaotic microcavities. So even if we start a test ray well above the critical angle χ_{cr}, at one point its angle of incidence will drop below χ_{cr} (the ergodicity theorem applies, at least for all practical purposes of a physicist, to chaotic ray billiards as well): The light ray can then refractively escape. How and where the refractive escape actually occurs, depends on a number of things: In case a stable periodic orbit exists with angles of incidence close to χ_{cr} and, in terms of the wave picture, hosts a mode, the emission pattern will be determined by the angles of incidence and the position of the reflection points of this orbit as, for example, observed for the so-called bow-tie (Gmachl et al. (1998)). If we think of a perturbed whispering gallery mode, the refractive escape will occur likely at boundary regions with higher curvature (Nöckel and Stone (1997)).

But this does not explain the directionality found in the Limaçon-shaped microdisk lasers where the laser output is actually by evanescent leakage (that ensures the high Q factors). Such a directional emission for Limaçon-shaped systems with deformation parameters ϵ around 0.4 and refractive indices n around 3.3 was first theoretically predicted (Wiersig and Hentschel (2008)) and, within one year, confirmed by four experimental groups (Song et al. (2009), Shinohara et al. (2009), Yan et al. (2009), Yi et al. (2009)). The agreement between the far fields obtained from wave computations and ray simulations, and the experimental results, respectively, is remarkable and depicted in Figure 2. To understand the reason behind this highly directional and robust far fields, the theory of chaotic dynamical systems is a useful tool that we will now use.

Chaotic rays and ray-wave correspondence

What eventually determines the far-field characteristics, is how all the test rays cross from the regime of total internal reflection to the region of refractive escape (one also says *how they cross the critical line* that separates these two regions). In the field of dynamical systems this is known as the *unstable manifold*, namely the collection of all unstable directions of our optical billiards: Our microdisk is nothing but a Hamiltonian system with two degrees of freedom. Its phase space is spanned by the arc length position along the system boundary and the sine of the angle of incidence for each reflection point. As usual (and without going much into the details), for each point in phase space we can find a stable (contracting) and unstable (expanding) direction, for example by starting a number of test rays in the vicinity of this point. The original shape of this point cloud will very soon be deformed and develop filaments. This has to happen because without the expanding direction there would be no chaos, and a contracting direction is then needed to fulfill Liouville's theorem, i.e. to conserve the phase space volume. One easily convinces oneself that it will be the unstable direction, i.e. the one in which the filaments grow such that they eventually cross the critical line, that determines the far-field characteristics of the microlaser.

This also explains the robustness of the far field: Whereas each individual ray reacts very sensitively to the slightest changes in the resonator shape (as characteristic for chaotic systems), the unstable manifold sort of averages over; and their filaments remain rather robust, an issue that is important and very helpful for applications. Nonetheless one can of course optimize the far-field characteristics via the unstable manifold by varying the resonator shape (keeping also in mind that a change in refractive index induces a change in the critical angle and the position of the critical line in phase space). And one has to keep in mind that, depending on the very specific details of your microcavity and its phase space, other mechanisms might come into the business, for example modification of transport through turnstiles, see for example Shim et al. (2011) or Yang et al. (2008).

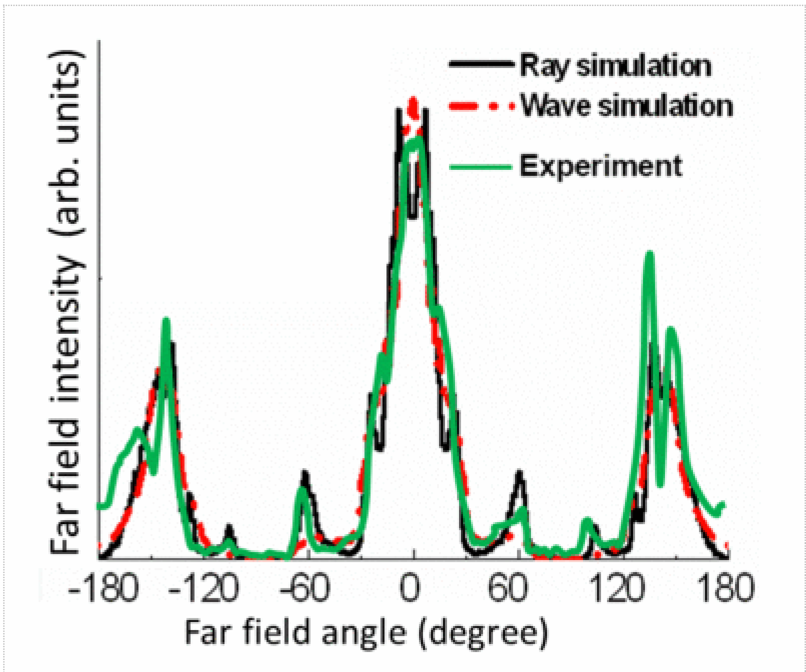

Figure 2: Far-field emission pattern of Limaçon-shaped microlasers.

Comparison of the results of ray simulations (Martina Hentschel, wave computations (Jan Wiersig), and experiments (Capasso group).

So far we have only talked about rays, but that's ok: The ray simulations agree remarkably well both with experimental data and wave simulations. This is quite exceptional and even more surprising when recalling that the wave simulations were done for rather small size parameters, i.e., well in the wave regime. This is an striking example for the power of the **concept of ray-wave correspondence**. It has proven to be a particularly useful and versatile tool in the investigation of chaotic microlasers and quantum chaos in optical and microwave billiards up to date (Schwefel et al. (2004), Bäcker et al. (2008), Shao et al. (2013), Dietz et al. (2014), Lafargue et al. (2014), Ryu et al. (2014), just to name a few out of many works here).

One remark for the experts in the field: Notice that the focus of wave simulations is typically on modes with high Q factors - an indication for evanescent light escape. However, the simulated light rays always escape by refraction. The way out of this apparent discrepancy is provided in phase space: The refracted light rays determine the unstable manifold - and it is precisely the unstable manifold that governs the evanescent leakage: The Husimi-function corresponding to the wave pattern intensities follows the filaments of the unstable manifold towards the critical line and determines, at the crossing points, the emission characteristics.

Alternatives without chaos and further remarks

When the focus is on directional far-field emission and not predominately on its origin in a chaotic microcavity laser, there are plenty of further possibilities to achieve lasing operation from, individual or coupled, microdisk systems. They include spiral cavities, composite systems, of course the above-mentioned VCSELs, and many other possibilities, see the review article on unidirectional light emission from ultralow-loss modes (Wiersig et al. (2011)) and references therein.

One last remark concerning the ray and wave simulations and their comparison with experiments: No amplification mechanisms have been taken into account in the ray and wave calculations presented here, nor are there semiclassical effects like the so-called Fresnel filtering or the Goos-Hänchen shift considered. Likewise, very often resonances of the *passive* cavity are used to compute the far fields. Interestingly, this works well for modes near the lasing threshold and for not to high pumping, whereas other methods like the Schrödinger-Bloch model have to be used when mode coupling via the gain medium and mode interaction become important (Harayama et al. (2005), Sunada et al. (2005), Shinohara et al. (2006)).

Eventually, a very brief remark on **Scars** is in order, a popular topic in the field of quantum chaos. Scars have been observed in optical microcavities and microlasers, and they promise to deepen our insight into these systems, e.g., by understanding why scarring seems to be more common here in comparison to hard wall quantum chaotic systems.

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Further reading

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External links

There is quite a number of groups worldwide working in the field of chaotic microlasers and optical microcavities in general. The following incomplete list (alphabetic) may serve as a starting point to get a broader view on the field:

K. An group (Seoul, Korea), H. Cao group (Yale, U.S.A.), F. Capasso group (Harvard, U.S.A.), T. Harayama group (Waseda university, Japan), M. Hentschel group (Ilmenau, Germany), Ch.-M. Kim group (Seoul, Korea), S.-W. Kim group (Busan, Korea), M. Lebenthal group (ENS Cachan, France), O. Legrand group (Nice, France), F. Mortessagne group (Nice, France), A. Richter group (Darmstadt, Germany), K. Sasaki group (Hokkaido University, Japan), H.-J. Stoeckmann group (Marburg, Germany), A. D. Stone group (Yale, U.S.A.), K. Vahala group (Caltech, U.S.A.), J. Wiersig group (Magdeburg, Germany), Y.-F. Xiao group (Peking, China), J. Zyss group (ENS Cachan, France).

See also

Microwave billiards and quantum chaos, Dynamical billiards, Quantum chaos

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